BASICS OF NUCLEAR MAGNETIC RESONANCE II

Bernard Lanz, PhD

CIBM MRI EPFL

27.02.2025

PHYS-770

CIBM translational MR neuroimaging & spectroscopy

THE ORIGIN OF MAGNETIC RESONANCE

- MRI/MRS are powerful imaging techniques for their <u>flexibility</u> and sensitivity to a wide range of tissue properties
- MRI/MRS are popular for their relative safety, their non-invasive nature, the use of <u>no ionizing radiation</u>.
- MRI/MRS are applications of NMR (nuclear magnetic resonance) to radiology.

(N) Nuclear (we play with the atom nucleus)
M Magnetic (we interact with it with magnetic fields)
R Resonance (we need to match the RF field to the natural precession of the nucleus)
I/S Imaging or Spectroscopy (the outcome measurements)

NUCLEAR MAGNETIC RESONANCE

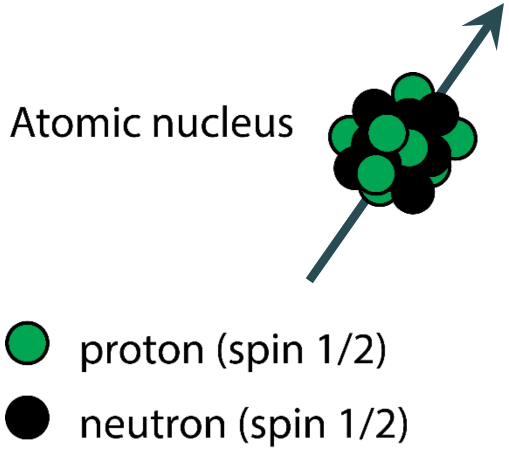
Magnetic fields in the game:

- Main magnetic field B_0 : on the order of 1 T
- Gradient fields G_x, G_y, G_z : on the order of 100 mT/m
- Radio frequency (resonant) field B_1: on the order of 10 µT
  - Very low amplitude (earth magnetic field ≈ 25-65 µT)
  - Very high frequency (on the order of 100 MHz)
  - Depends linearly on the applied B_0 field and Nucleus of interest

THE ORIGIN OF MAGNETIC RESONANCE

The dual nature of the MR phenomenon

Interaction of magnetic moments of nuclei with the magnetic field:



- Atomic nuclei possess an intrinsic angular momentum (spin) L, as a composition of the spin of their composing protons and neutrons

- According to quantum mechanics, L is quantized: $|\vec{L}| = \left(\frac{h}{2\pi}\right)\sqrt{I(I+1)}$
  I= 0, 1/2, 1, 3/2, ...

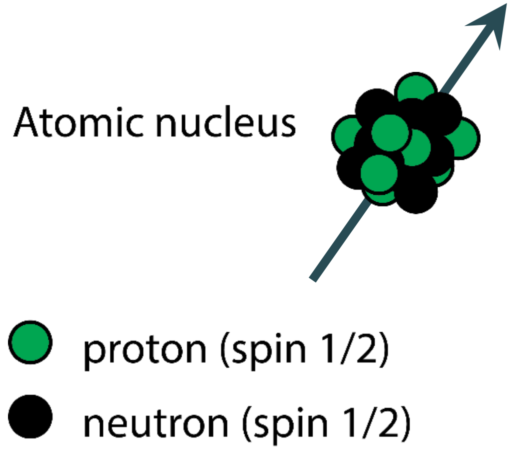
Depending on their composition:

- even number of protons and neutrons	I=0
- odd number of protons and neutrons	I=integer
- odd number of protons and even number of neutrons	I=half-integer
- odd number of neutrons and even number of protons	I=half-integer

NUCLEAR SPIN AND MAGNETIC MOMENT

The dual nature of the MR phenomenon

Interaction of magnetic moments of nuclei with the magnetic field:



- The intrinsic angular momentum L is associated to a magnetic moment μ:

$$\vec{\mu} = \gamma \vec{L}$$

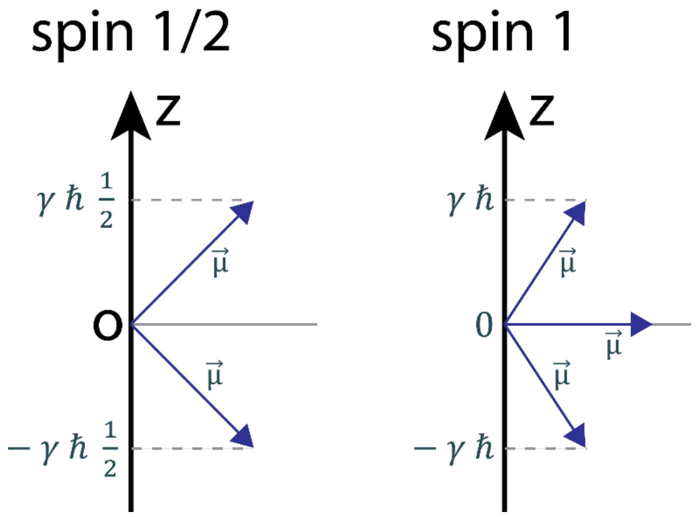
  with γ the gyromagnetic ratio, specific for each nucleus, specifying the strength of coupling between the magnetic field and the angular momentum.

- In the magnetic field $\overrightarrow{B_0}$ defined as $\overrightarrow{B_0} = B_0 \, \hat{e}_z$, the measured projection of the nuclear magnetic moment $\vec{\mu}$ are quantized:

$$\mu_z = \gamma \hbar m$$

with $m = -$I, -I+1, …,I-1, I

(2I+1 values)



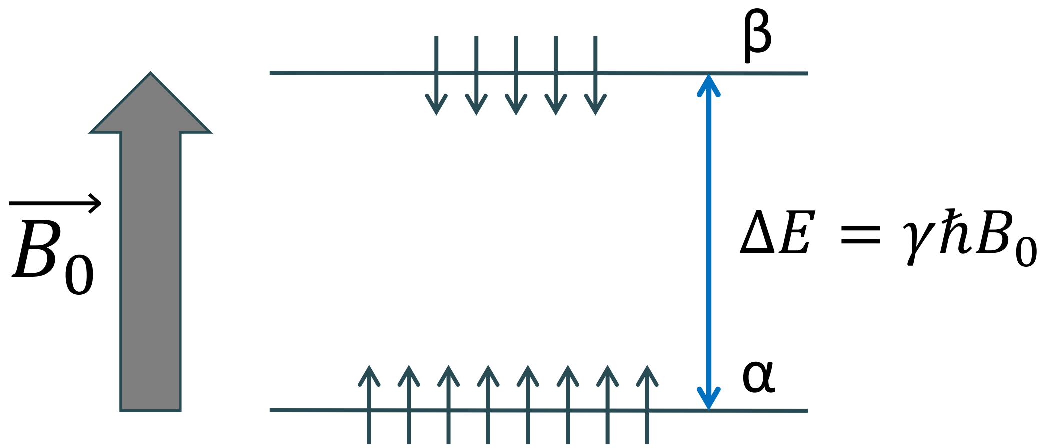
ZEEMAN ENERGY LEVELS

- Zeeman energy:
  Potential energy of a magnetized nucleus in an external magnetic field

$$E = -\vec{\mu} \cdot \overrightarrow{B_0}$$

$$E = -\mu_z B_0 = -\gamma\, \hbar\, \mathrm{m}\, B_0$$

For a spin ½ ($m = \pm 1/2$)

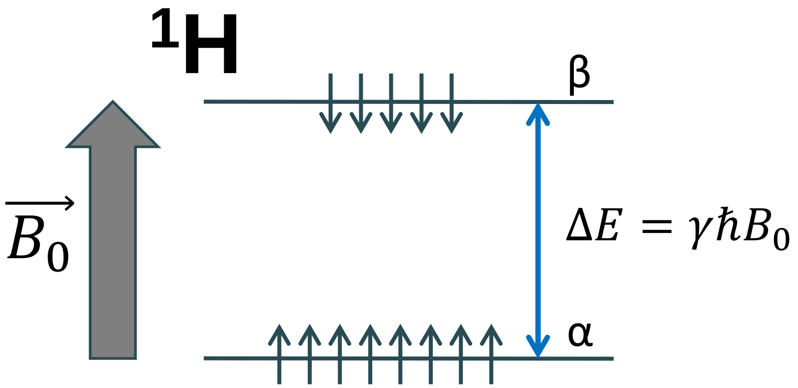


There is no spontaneous transition between energy levels
(no emission of energy)

All changes in the energy of nuclei are through stimulated transitions

ZEEMAN ENERGY LEVELS: SPIN DISTRIBUTION

- Zeeman energy:



Boltzmann distribution:

$$\left(\frac{n_\alpha}{n_\beta}\right) = e^{\frac{\gamma B_0 \hbar}{kT}}$$

Linearized Boltzmann distribution: $$(n_\alpha - n_\beta) \approx \left(\frac{nh\gamma B_0}{2kT}\right)$$

- Amplitude of the magnetization: $M_0 = (n_\alpha - n_\beta)\mu_z = (n_\alpha - n_\beta)\,\gamma\,\frac{\hbar}{2}$

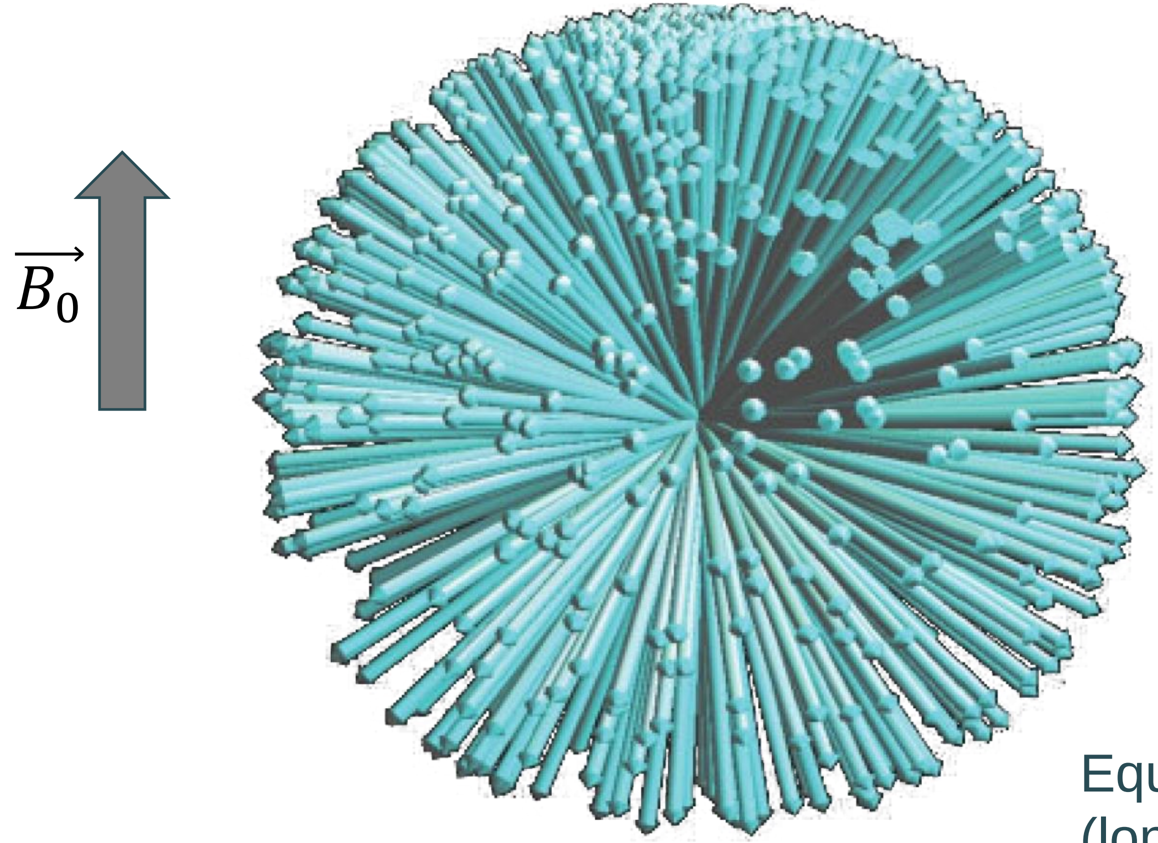
- Total magnetization: $M_0 = n\,\frac{1}{2}(\gamma\hbar)^2 \left(\frac{B_0}{2kT}\right) \propto \gamma^2$

INTERESTING NUCLEI / ISOTOPES

Nucleus	Magnetic moment	Gyromagnetic ratio (rad·MHz T^{-1})	Relative sensitivity	Natural abundance (%)
^{1}H	**1/2**	**267.522**	**1.0**	**99.98**
^{2}H	1	41.066	0.00965	0.015
^{13}C	1/2	67.283	0.0159	1.108
^{14}N	1	19.338	0.0101	99.63
^{15}N	1/2	−27.126	0.0104	0.37
^{19}F	1/2	251.815	0.83	100
^{23}Na	3/2	70.808	0.0925	100
^{29}Si	1/2	−53.190	0.00784	4.7
^{31}P	1/2	108.394	0.0663	100

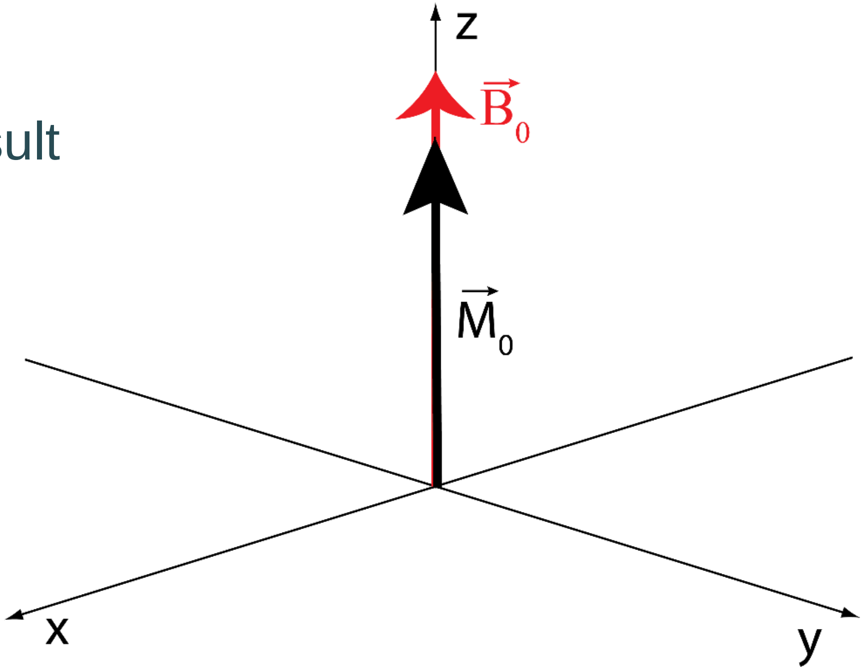
MACROSCOPIC MAGNETIZATION

Large spin ensemble



Macroscopic result

Magnetization



Equilibrium magnetization M_0 is parallel with B_0 (longitudinal magnetization),
its status can be changed by a radiofrequency magnetic field

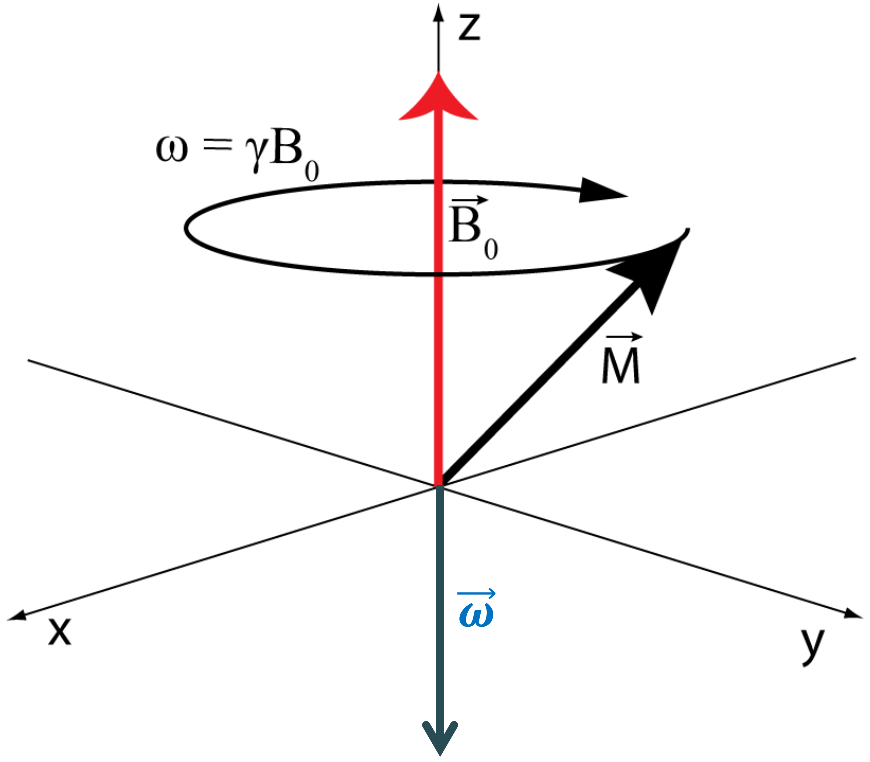
PRECESSION

<u>Equation of precession:</u>

$$\frac{d\vec{M}}{dt} = \gamma \vec{M} \times \vec{B_0}$$

For positive γ (^1H, ^{13}C, ^{31}P)

($\vec{\omega}//\vec{B_0}$ for negative γ, ^{15}N, ^{17}O)

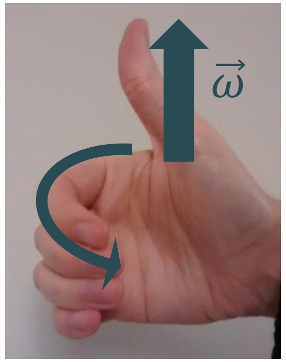


<u>Rotation defined with an angular speed vector:</u>

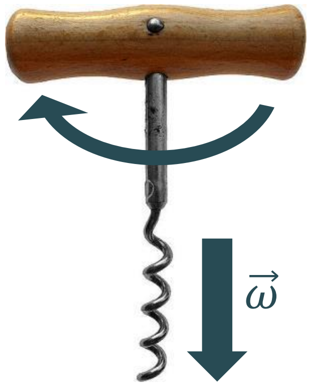
$$\frac{d\vec{M}}{dt} = \vec{\omega} \times \vec{M}$$

With

$$\omega = \gamma B_0 \equiv \omega_0$$ The Larmor frequency



Or corkscrew rule…



THE ROTATING FRAME(S)

LARMOR FRAME

In the laboratory frame:

$$\frac{d\vec{M}}{dt} = \gamma \vec{M} \times \vec{B_0}$$

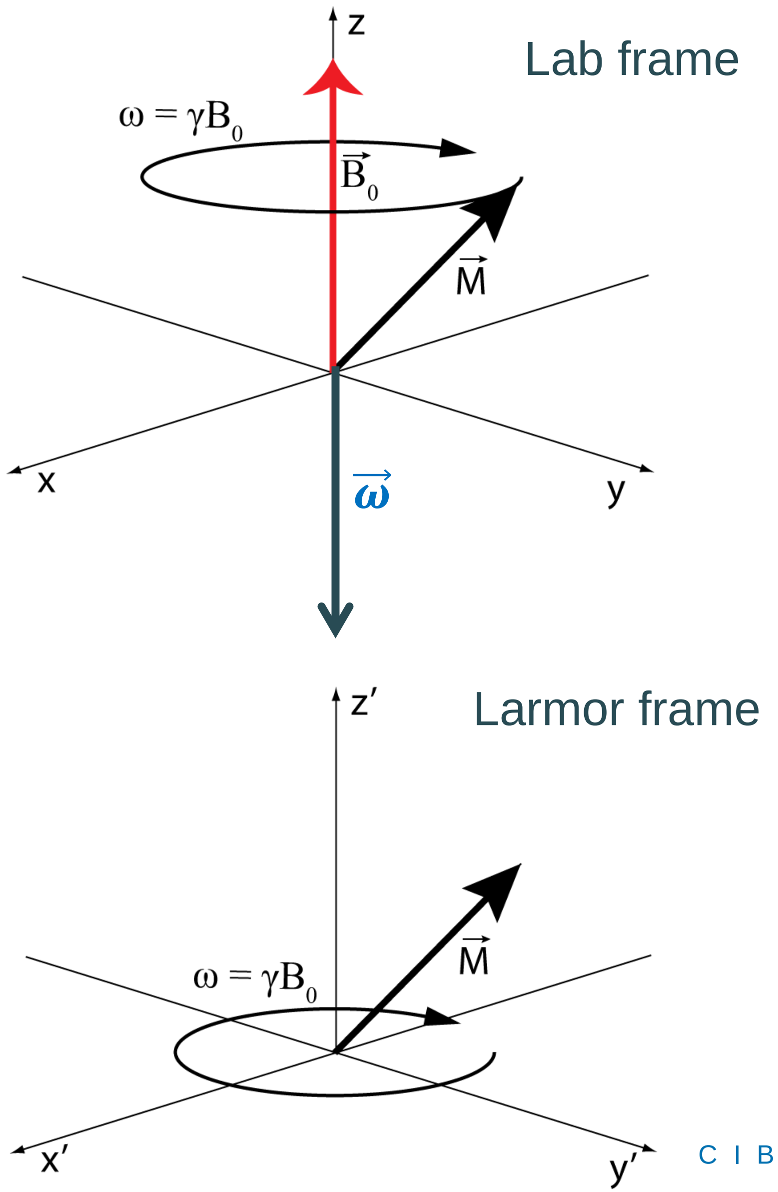
Expression in the rotating frame:

$$\left(\frac{d\vec{M}}{dt}\right)_{lab} = \left(\frac{d\vec{M}}{dt}\right)_{rot} + \vec{\omega} \times \vec{M}$$

In the Larmor rotating frame:

$$\left(\frac{d\vec{M}}{dt}\right)_{rot} = \left(\frac{d\vec{M}}{dt}\right)_{lab} + \gamma \vec{B_0} \times \vec{M}$$

$$\frac{d\vec{M}}{dt} = 0$$ → No apparent B_0 magnetic field

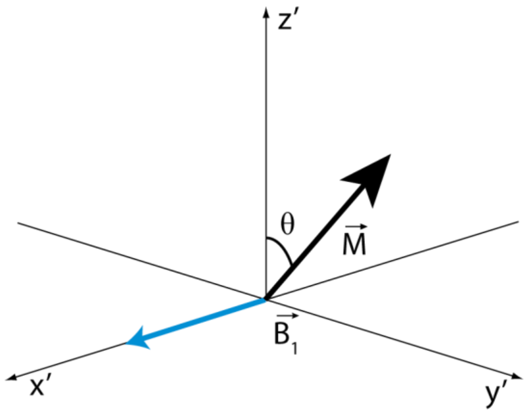


FLIP ANGLE

THE RESONANCE CONDITION

If we apply a pulse $B_1(t)$ right at the Larmor frequency of the considered spin system

$$\rightarrow \quad \omega_{RF} = \omega_0$$

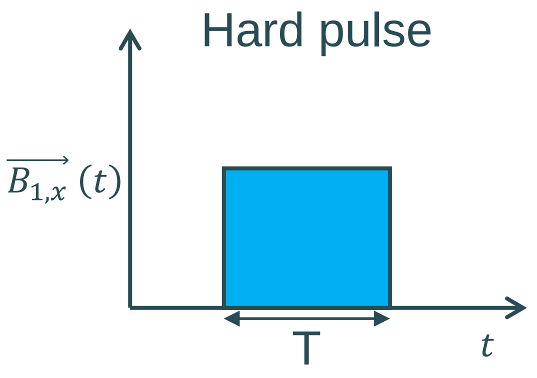


The dynamics of the magnetization is very much simplified:

$$\left(\frac{d\vec{M}}{dt}\right)_{rot} = \gamma \vec{M} \times \overrightarrow{B_1(t)}$$ with $B_1(t)$ the envelope of the pulse

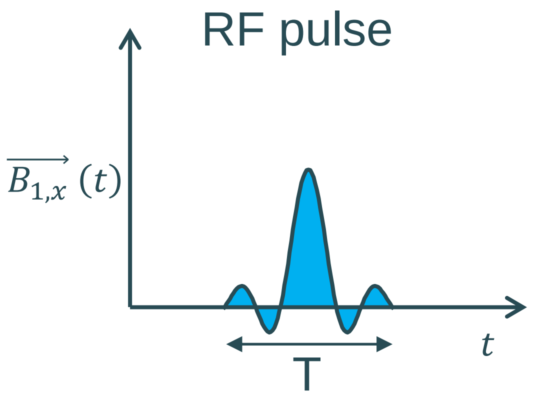
Flip angle (nutation angle)

$$\theta = \gamma \int_t^T B_1(t)dt$$



For a hard pulse

$$\theta = \gamma B_1\, T$$



For a general pulse

$$\theta = \gamma B_1\, T\, S_{int}$$

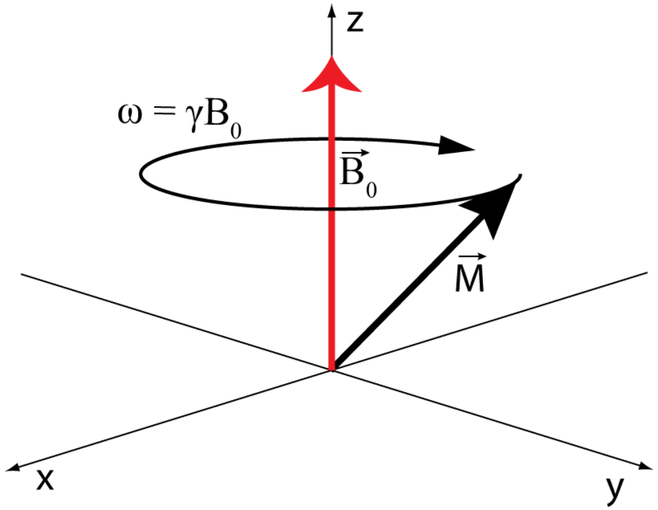
with S_{int} the pulse shape integral

QUANTUM DESCRIPTION

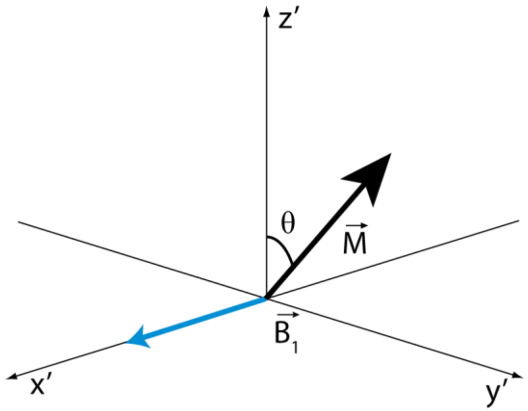
Single spin expectation values

Haake et al., Magnetic Resonance Imaging, Physical principles and sequence design, Wiley 1999

$$
\begin{aligned}
<\mu_x> &= \frac{\gamma\hbar}{2}\sin\theta\cos(\phi_0 - \omega_0 t)\\
<\mu_y> &= \frac{\gamma\hbar}{2}\sin\theta\sin(\phi_0 - \omega_0 t)\\
<\mu_z> &= \frac{\gamma\hbar}{2}\cos\theta
\end{aligned}
$$

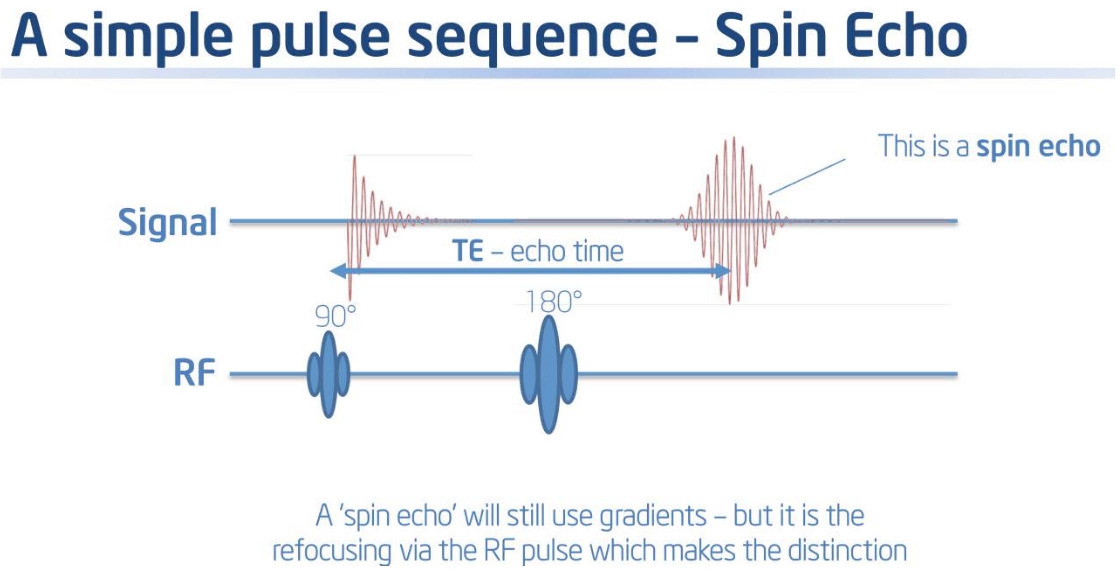


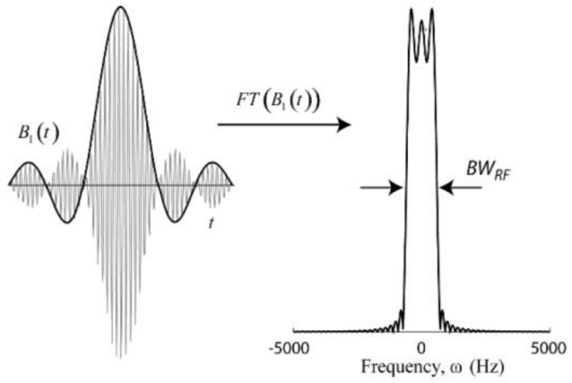
$$
\begin{aligned}
\langle\mu_{x'}(t)\rangle &= \langle\mu_{x'}(0)\rangle\\
\langle\mu_{y'}(t)\rangle &= \langle\mu_{y'}(0)\rangle\cos\omega_1 t + \langle\mu_z(0)\rangle\sin\omega_1 t\\
\langle\mu_z(t)\rangle &= -\langle\mu_{y'}(0)\rangle\sin\omega_1 t + \langle\mu_z(0)\rangle\cos\omega_1 t
\end{aligned}
$$

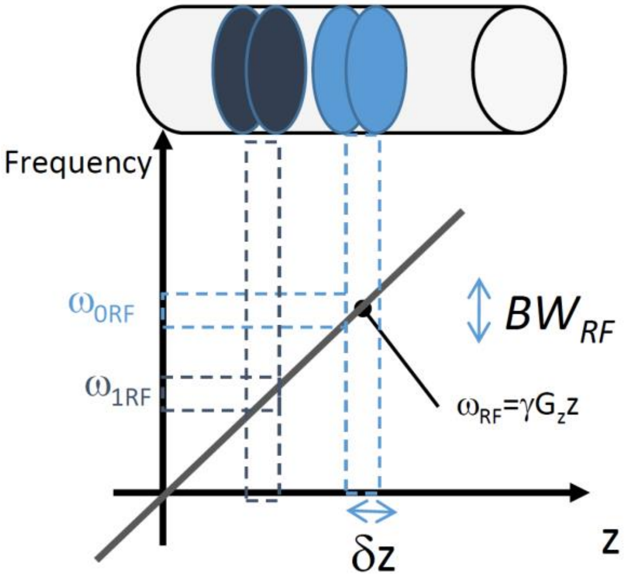


RF PULSES ARE NEEDED FOR ALL MRI / MRS ACQUISITIONS

For slice selection, a gradient is applied during an RF-pulse, which is characterized by its excitation bandwidth (BW_{RF})







THE ROTATING FRAME(S)

RF FRAME

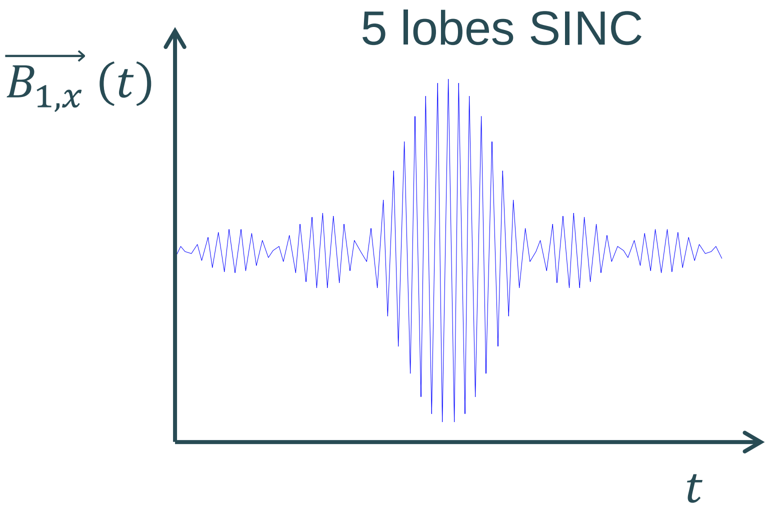
In the laboratory frame:

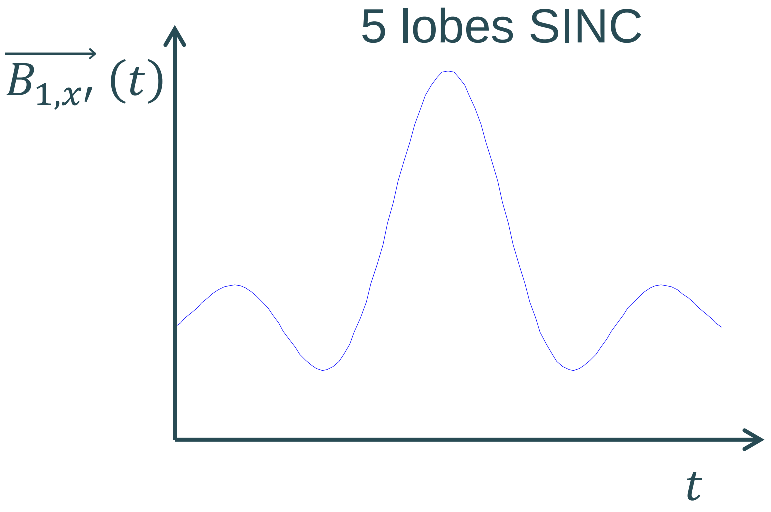
$$\overrightarrow{B_1}(t) = B_1(t)\cos(\omega_{RF}\, t)\,\hat{x} + B_1(t)\sin(\omega_{RF}\, t)\,\hat{y}$$

In the RF rotating frame:

$$\begin{pmatrix} B_{1,x'}(t) \\ B_{1,y'}(t) \\ B_{1,z'}(t) \end{pmatrix}_{rot} = \begin{pmatrix} \cos(\omega_{RF}\, t) & \sin(\omega_{RF}\, t) & 0 \\ -\sin(\omega_{RF}\, t) & \cos(\omega_{RF}\, t) & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} B_1(t)\cos(\omega_{RF}\, t) \\ B_1(t)\sin(\omega_{RF}\, t) \\ 0 \end{pmatrix}_{lab}$$

$$\begin{pmatrix} B_{1,x'}(t) \\ B_{1,y'}(t) \\ B_{1,z'}(t) \end{pmatrix}_{rot} = \begin{pmatrix} B_1(t) \\ 0 \\ 0 \end{pmatrix}$$



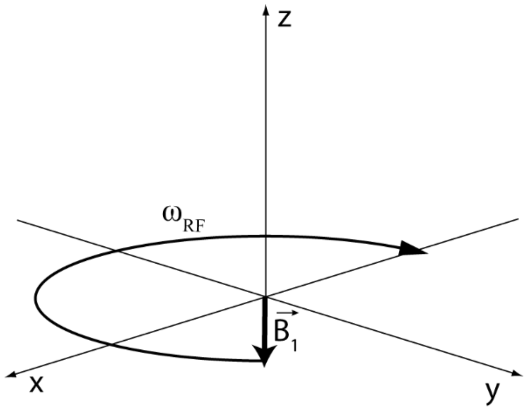


The rotating frame has demodulated the RF oscillation and transformed the rapidly oscillating RF field into a much simpler form, the time-dependent envelope $B_1(t)$.

THE ROTATING FRAME(S)

RF FRAME

Lab frame



In the laboratory frame:

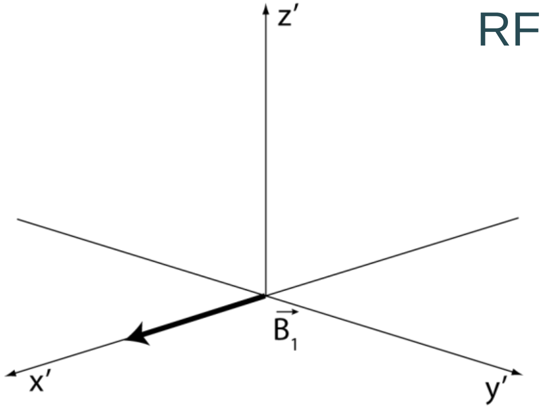
$$\overrightarrow{B_1}(t) = B_1(t)\cos(\omega_{RF}\,t)\,\hat{x} + B_1(t)\sin(\omega_{RF}\,t)\,\hat{y}$$

In the RF rotating frame:

$$\begin{pmatrix} B_{1,x'}(t) \\ B_{1,y'}(t) \\ B_{1,z'}(t) \end{pmatrix}_{rot} = \begin{pmatrix} \cos(\omega_{RF}\,t) & \sin(\omega_{RF}\,t) & 0 \\ -\sin(\omega_{RF}\,t) & \cos(\omega_{RF}\,t) & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} B_1(t)\cos(\omega_{RF}\,t) \\ B_1(t)\sin(\omega_{RF}\,t) \\ 0 \end{pmatrix}_{lab}$$

$$\begin{pmatrix} B_{1,x'}(t) \\ B_{1,y'}(t) \\ B_{1,z'}(t) \end{pmatrix}_{rot} = \begin{pmatrix} B_1(t) \\ 0 \\ 0 \end{pmatrix}$$

RF frame



The rotating frame has demodulated the RF oscillation and transformed the rapidly oscillating RF field into a much simpler form, the time-dependent envelope $B_1(t)$.

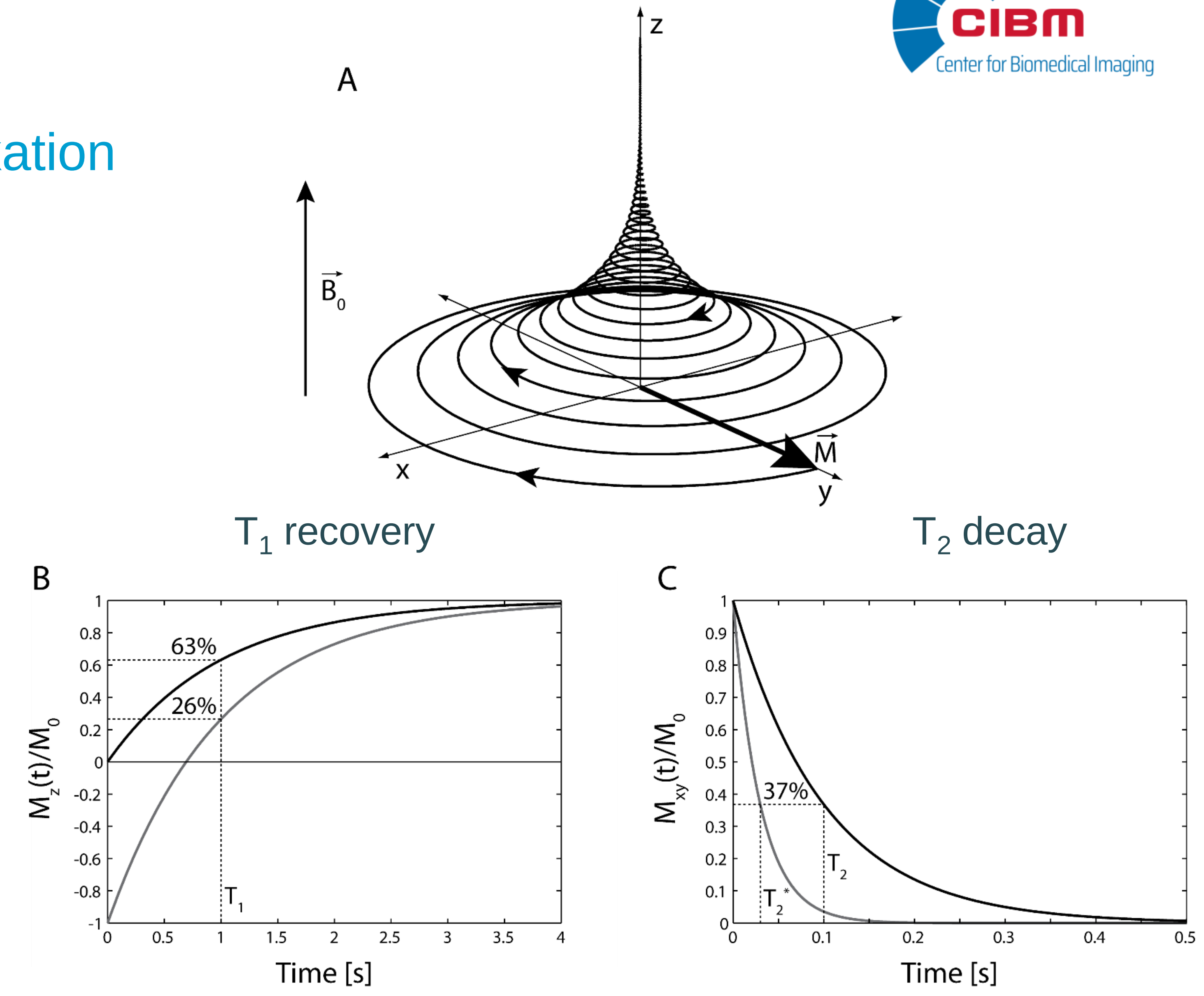
RELAXATION(S)

Macroscopic magnetization relaxation

After excitation, the spin ensemble reaches its thermal equilibrium:

1. spin-lattice relaxation time T_1
   (transition from higher energy state to equilibrium)
   → longitudinal relaxation

2. spin-spin relaxation time T_2
   (loss of coherence (order) of the microscopic components)
   → transverse relaxation



Relaxation is caused by random fluctuating magnetic fields on a molecular or submolecular level (dominated by dipolar coupling)

TRANSVERSE RELAXATION

Macroscopic contributions

- Inhomogeneous B_0 contributes to the decay of transverse magnetization

(bad shim, static B_0 components due to differences in magnetic susceptibility between water and air leads in the proximity to their boundary)

- Then the decay of M_{xy} is faster (with the time constant $1/T_2^*$) than that corresponding to the T_2 relaxation time:

$$\frac{1}{T_2^*} = \frac{1}{T_2} + \Delta B_0 / 2\gamma$$

ΔB_0 increases with magnetic field (shorter T_2^*) and has less impact on low gyromagnetic ratio nuclei

T_1 and particularly T_2 relaxation times are shorter than in aqueous solutions

due to Interactions water / biomacromolecules / low molecular-weight solutes

BLOCH EQUATION

$$\frac{d\vec{M}}{dt} = \underbrace{\gamma \vec{M} \times \vec{B}}_{} \underbrace{- \frac{M_x\hat{x} + M_y\hat{y}}{T_2} - \frac{(M_0 - M_z)\hat{z}}{T_1}}_{} + \underbrace{D\nabla^2 \vec{M}}_{}$$

Larmor precession
With the <u>total field</u>
(or effective field in the rotating frame)

Transverse and longitudinal Relaxations

T2 ~ 50-100 ms
T1 ~ 1 s

Pulse duration ~ 100 µs – 5 ms

Typically Neglected for RF design

Diffusion effects.

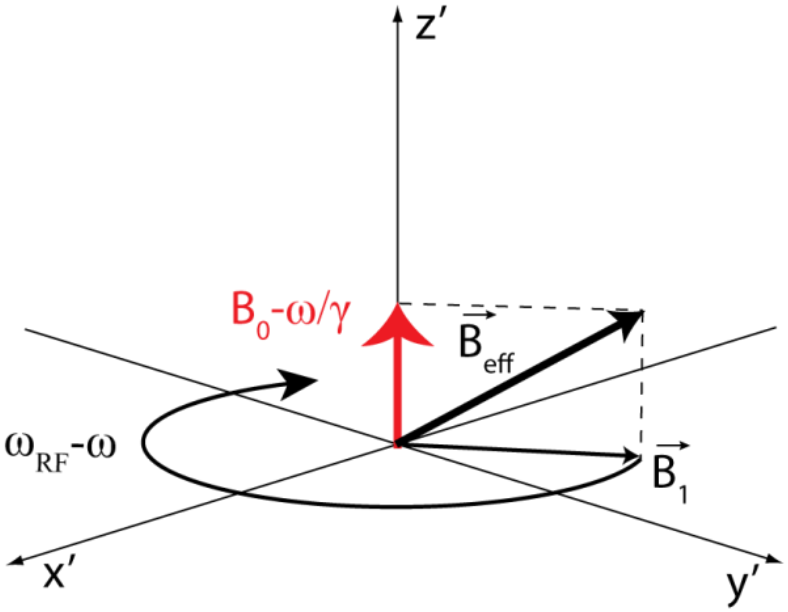
Typically Neglected for RF design

BLOCH EQUATION

IN A ROTATING FRAME (ω)

$$\left(\frac{d\vec{M}}{dt}\right)_{rot} = \gamma \vec{M} \times \overrightarrow{B_{eff}}$$

$$B_{eff} = B_1(t)\cos((\omega_{RF}-\omega)\,t)\,\hat{x} + B_1(t)\sin((\omega_{RF}-\omega)t)\,\hat{y} + \left(B_0 - \frac{\omega}{\gamma}\right)\hat{z}$$

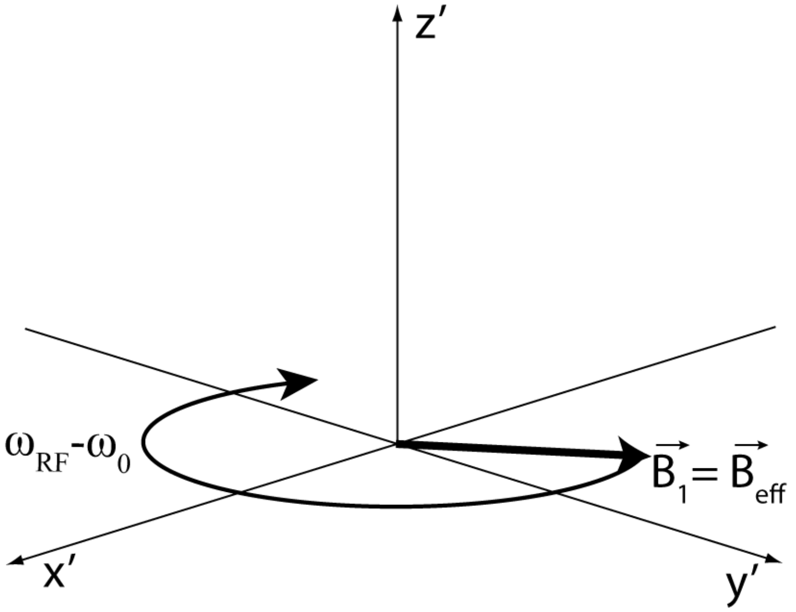


BLOCH EQUATION

IN THE LARMOR ROTATING FRAME (ω_0)

$$\left(\frac{d\vec{M}}{dt}\right)_{rot} = \gamma\vec{M} \times \overrightarrow{B_{eff}}$$

$$B_{eff} = B_1(t)\cos((\omega_{RF}-\omega_0)\,t)\,\hat{x} + B_1(t)\sin((\omega_{RF}-\omega_0)t)$$

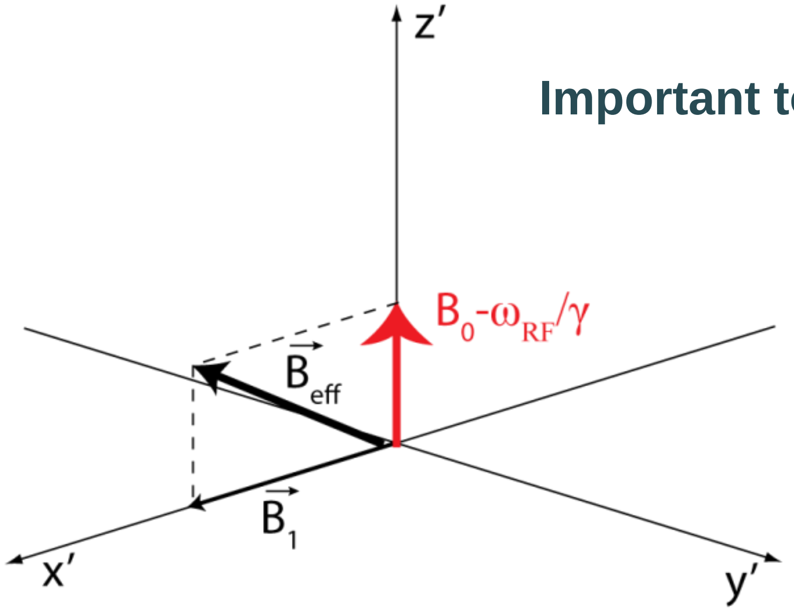


BLOCH EQUATION

IN THE RF FRAME (ω_{RF})

$$\left(\frac{d\vec{M}}{dt}\right)_{rot} = \gamma\vec{M} \times \overrightarrow{B_{eff}}$$

$$B_{eff} = B_1(t)\,\hat{x} + \left(B_0 - \frac{\omega_{RF}}{\gamma}\right)\hat{z}$$



Important to consider for off-resonance effects

FLIP ANGLE

OFF-RESONANCE EFFECTS

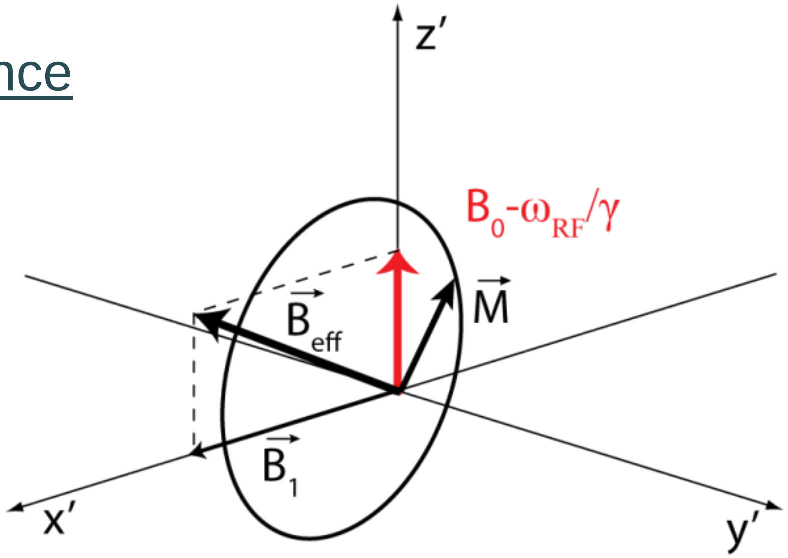
In the RF rotating frame:

$$\left(\frac{d\vec{M}}{dt}\right)_{RF} = \gamma\vec{M} \times \left[B_1(t)\,\hat{\mathrm{x}} + \left(B_0 - \frac{\omega_{RF}}{\gamma}\right)\hat{\mathrm{z}}\right]$$

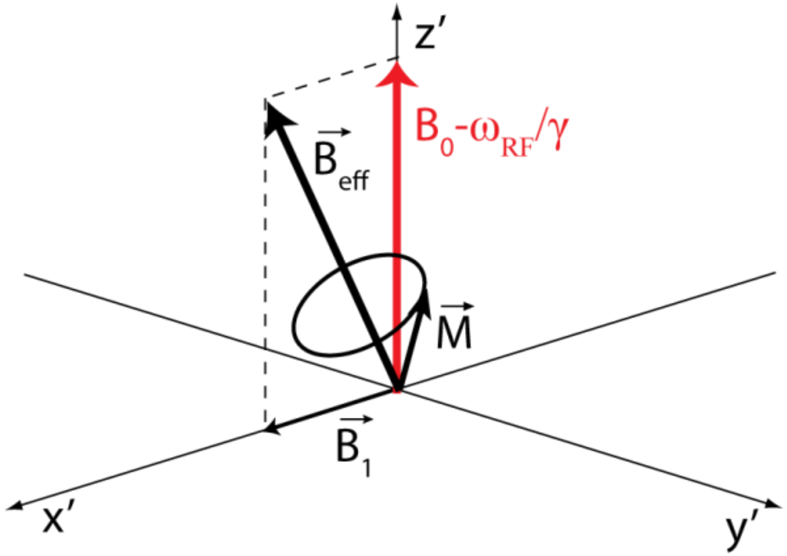
With the offset $\Delta\omega = \gamma B_0 - \omega_{RF}$

The magnetization of a spin system with same offset $\Delta\omega$ is called an isochromat.

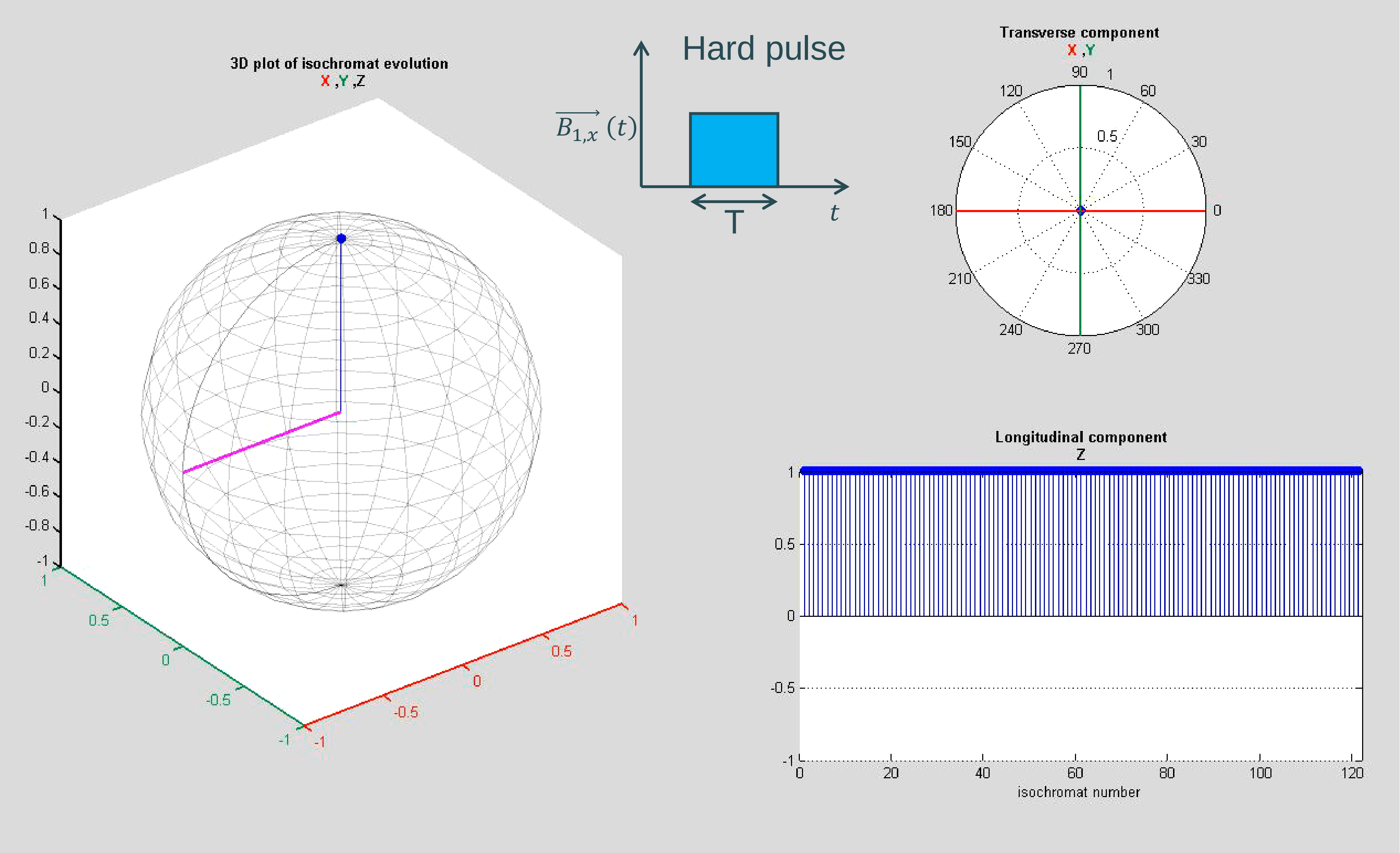
Off-resonance



Far off-resonance
(no xy plane
crossing anymore)



PULSE BANDWIDTH (90° PULSE EXAMPLE)



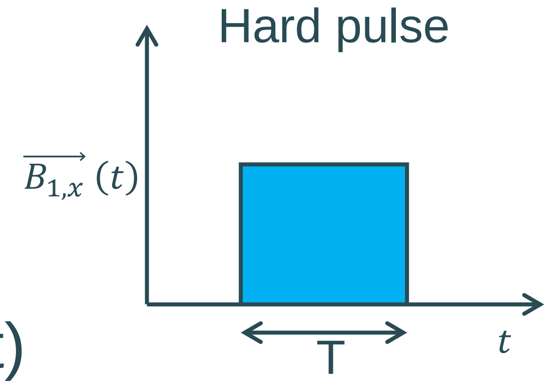
FLIP ANGLE

PULSE BANDWIDTH (180° PULSE EXAMPLE)

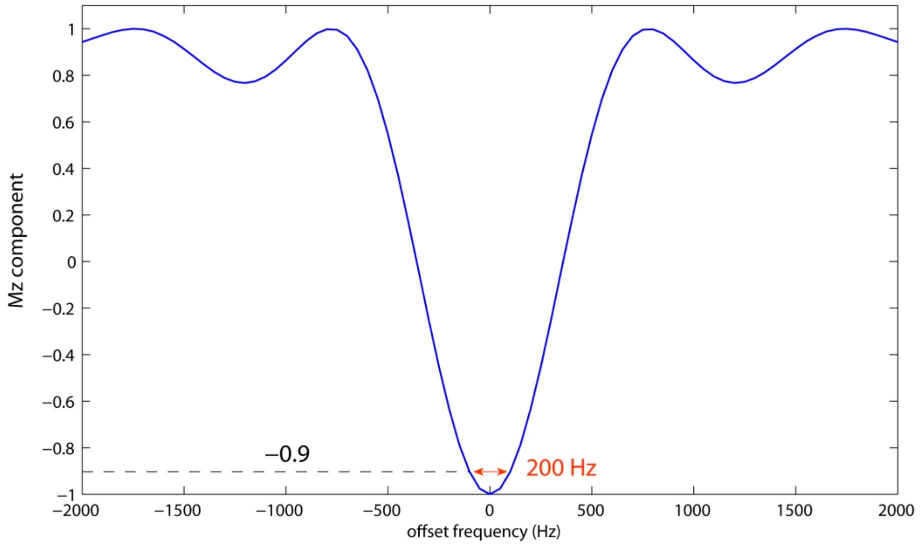
Hard pulse:

Parameters:

$B_1(t)$
T
ω_{RF} or ω_{RF}(t)

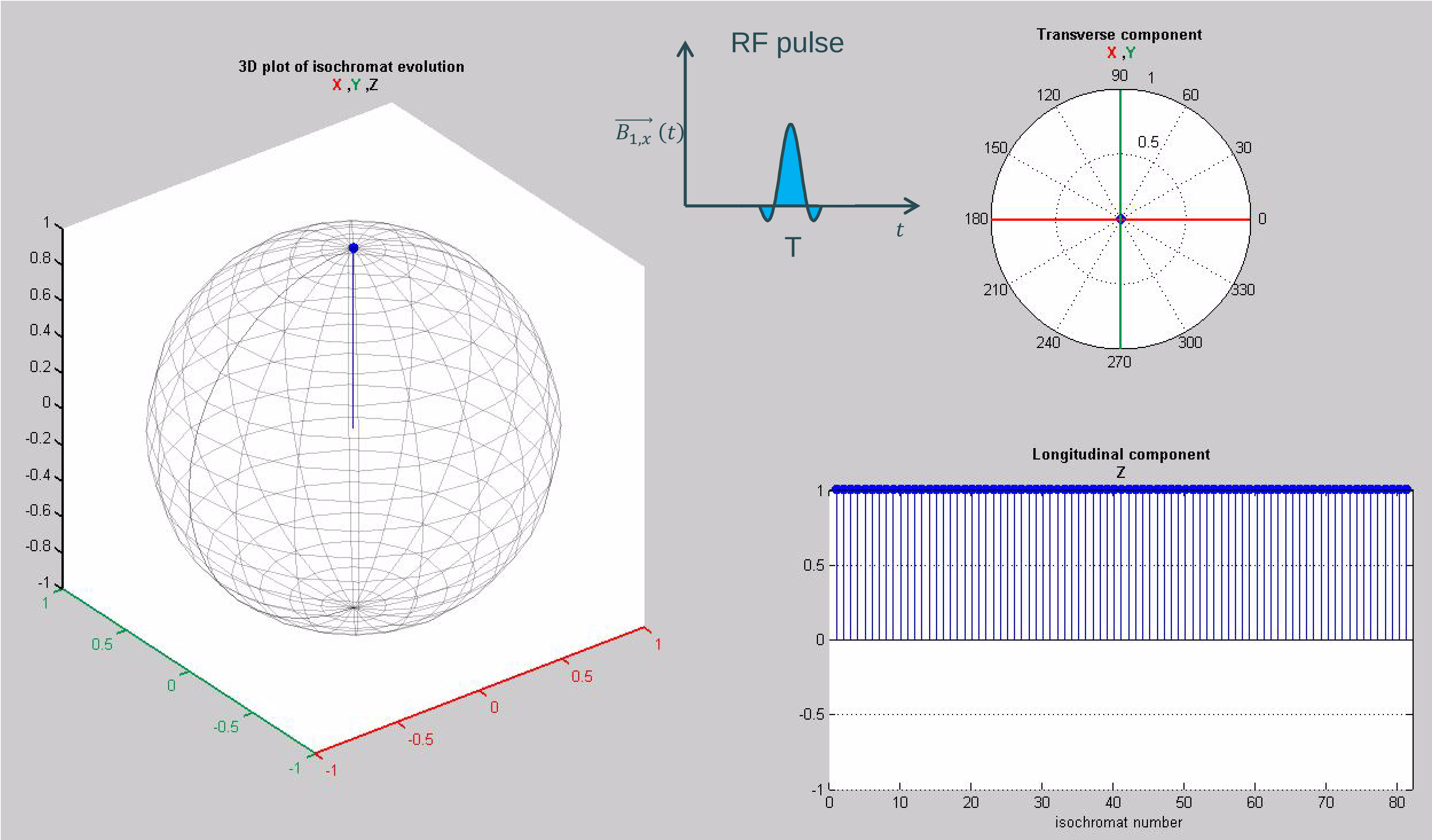


Characteristics:
(Example: inversion pulse)



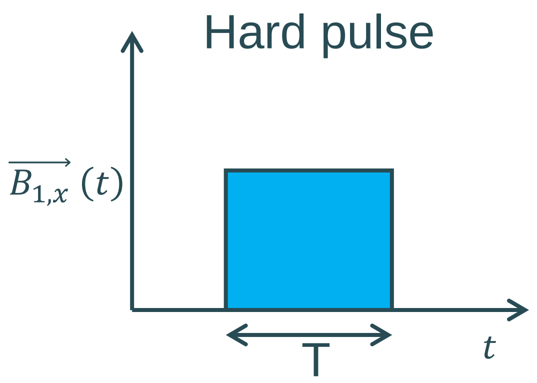
SINC PULSES

90° B1 at 32.8 μT → duration 3.23 ms



FLIP ANGLE

FOURIER TRANSFORM APPROXIMATION



<u>Complex notation:</u> $M_\perp = M_x + i\, M_y$

precession: $\frac{dM_\perp}{dt} = -i\Delta\omega M_\perp + i\gamma B_1(t) M_z(t)$

With the initial conditions $M_\perp(0) = 0$

$$M_\perp(\mathrm{t}, \Delta\omega) = i\gamma \mathrm{e}^{-\mathrm{i}\,\Delta\omega\, t} \int_0^t M_z(t')B_1(t')e^{i\,\Delta\omega t'}\, dt'$$

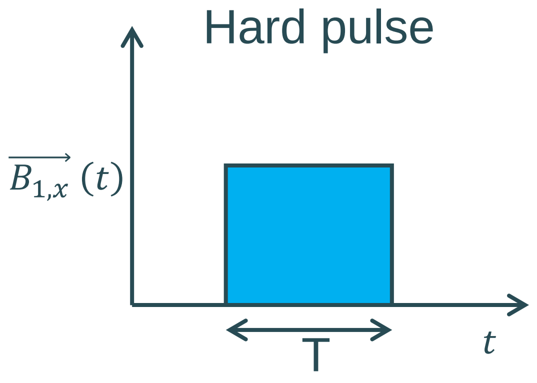
For small flip angles, $M_z(t') = M_0$

$$|M_\perp(\mathrm{t}, \Delta\omega)| = \gamma M_0 \left| \int_0^t B_1(t')e^{i\,\Delta\omega t'}\, dt' \right|$$

<u>**For small flip angles**</u>, the pulse response profile is approximately equal to the modulus of the inverse Fourier transform of the RF envelope.

FLIP ANGLE

FOURIER TRANSFORM APPROXIMATION

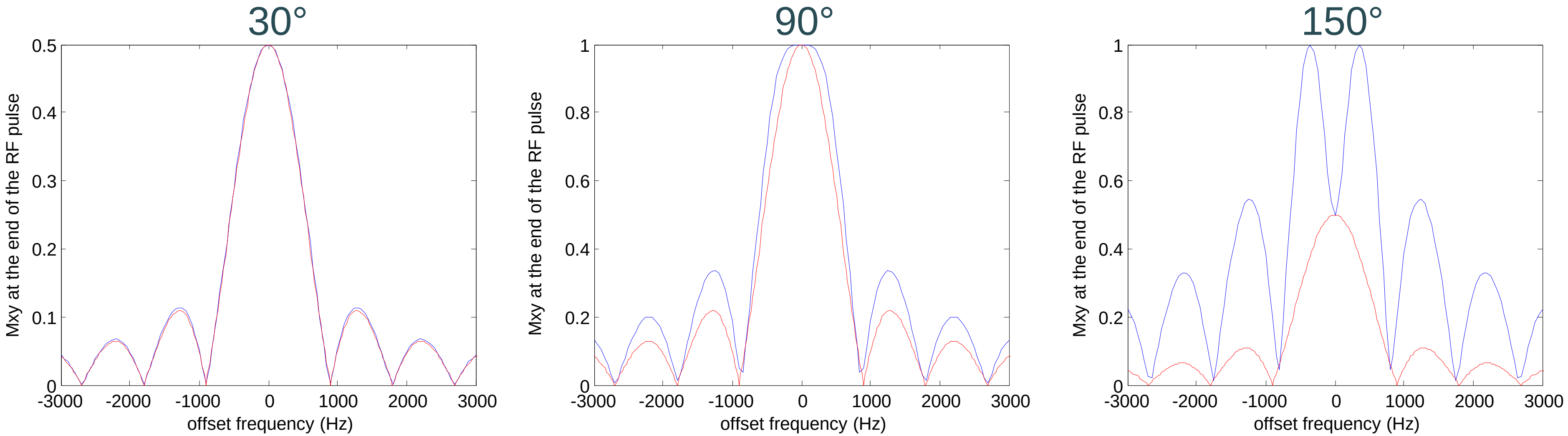


Complex notation: $M_{\perp} = M_x + i\, M_y$

$$|M_{\perp}(\mathrm{t}, \Delta\omega)| = \gamma M_0 \left| \int_0^t B_1(t') e^{i\, \Delta\omega t'}\, dt' \right|$$

inverse Fourier transform

Different angles:

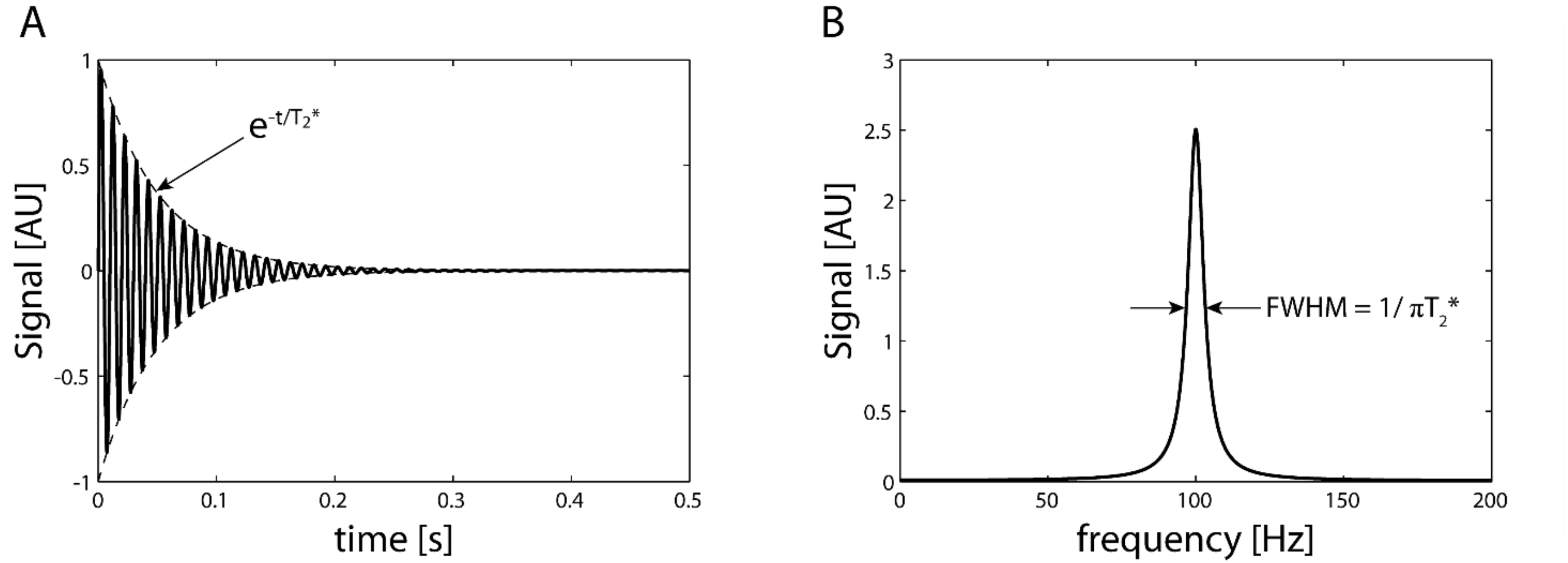


Fourier transform of the envelope
Solution of the Bloch equation

BASIC PULSE SEQUENCES

(FID, FREE INDUCTION DECAY)

Pulse acquire sequence



During TR, T1 relaxation takes place.

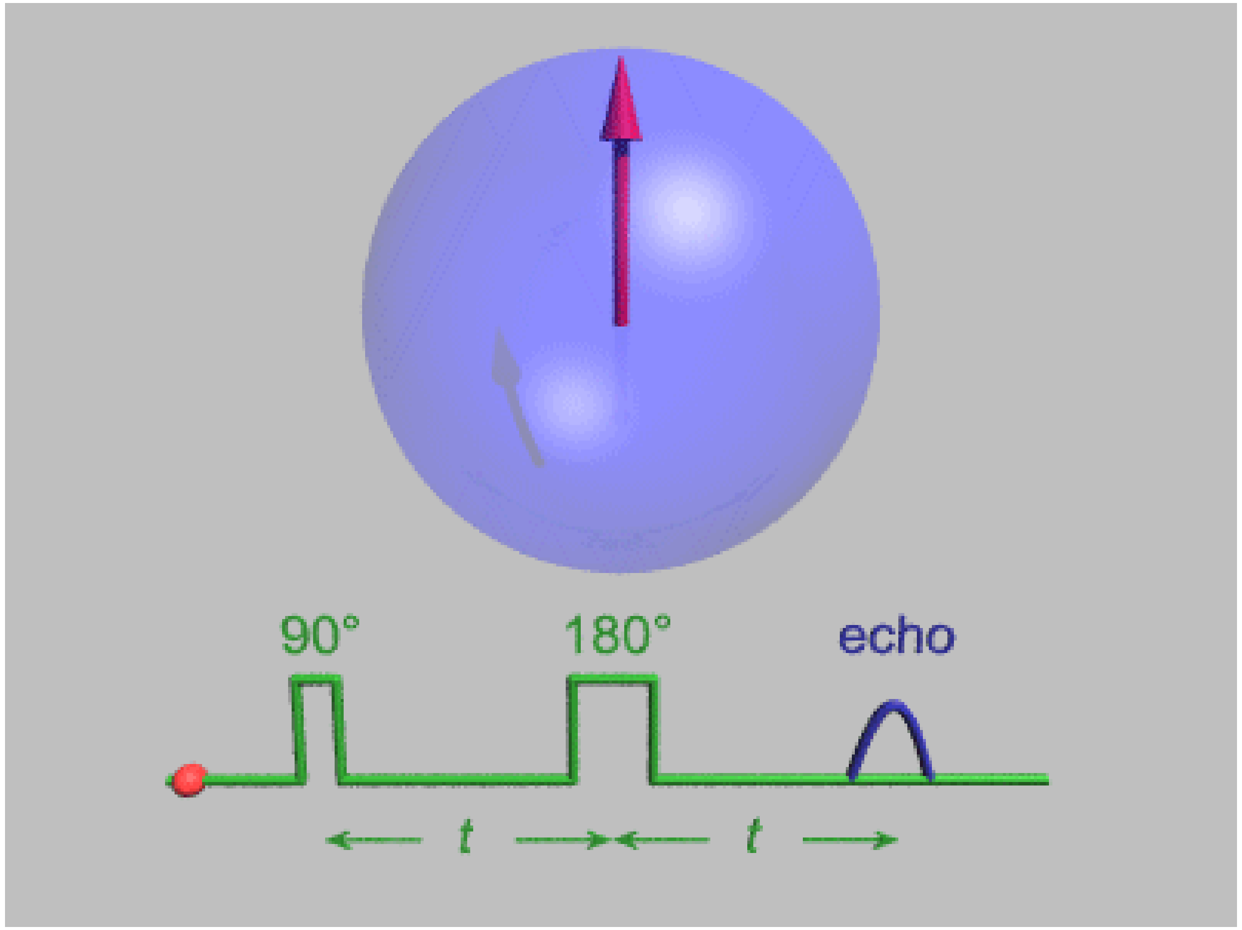
There is an optimal flip angle for maximal signal for a given TR and T1:

$$\theta_E = arccos\left(e^{-\frac{TR}{T_1}}\right)$$

(Ernst angle)

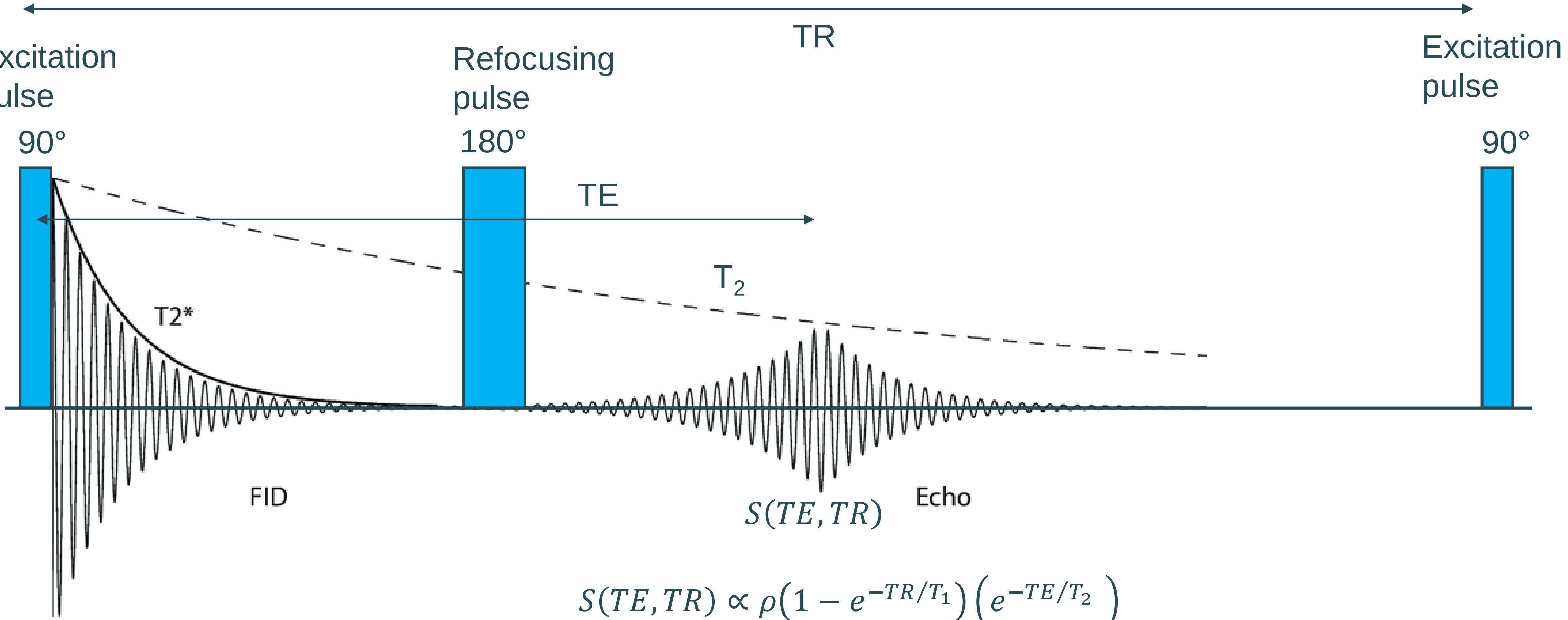
BASIC PULSE SEQUENCES (SPIN-ECHO)

Spin gymnastics



Gavin W Morley, Wikimedia Commons

BASIC PULSE SEQUENCES (SPIN-ECHO)



$$S(TE,TR) \propto \rho\left(1 - e^{-TR/T_1}\right)\left(e^{-TE/T_2}\right)$$

In MRI, this can be used to generate T_2 and T_1 contrasts.
In MRS, it is typically used in fully relaxed conditions (T_1 > 5 TR) to recover in-phase signal

THANK YOU FOR YOUR ATTENTION

Questions ?

PHYS-770

bernard.lanz@epfl.ch